# THE EFFECT OF DEFORMATION ON THE EFFECTIVE CONDUCTIVITY OF A DILUTE SUSPENSION OF DROPS IN THE LIMIT OF LOW PARTICLE PECLET NUMBER

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Abstract—The effective thermal conductivity of a dilute suspension of slightly deformed droplets is calculated in the limit of small particle Peclet number for the undisturbed bulk shear,  $u = \gamma y$ , and the linear bulk temperature distribution,  $T = \alpha y$ . Two distinct cases of small deformation are considered; deformation dominated by interfacial tension forces, and deformation dominated by viscous forces in the drop. The results show that the presence of deformation can cause a fundamental change in form of the dominant, flow-induced contribution to the effective conductivity.

#### INTRODUCTION

Recently, Leal (1973) considered the effective conductivity of a dilute suspension of neutrally buoyant spherical drops in the limit of low particle Peclet number for the case of a simple bulk shear flow  $(u = \gamma y)$  and a linear bulk temperature distribution  $(T = \bar{\alpha} y)$ . A general expression was presented relating the effective (bulk) conductivity of the suspension to the microscale velocity and temperature fields associated with each individual particle. Using this relationship, the effective conductivity was evaluated for  $Re \ll Pe \ll 1$ , with the velocity fields obtained from the classical creeping flow solution of Taylor (1932) for a spherical drop in shear flow, and the microscale temperature field calculated using the method of matched asymptotic expansions.

It is clear, of course, that no droplet in a real suspension will be exactly spherical, and, indeed, the deviations from spherical shape can often become quite large. Nevertheless, it would normally be expected that the solution for a sphere should provide an adequate first estimate for the effective conductivity in a flowing suspension, provided only that either the surface tension or droplet viscosity is sufficiently large so that the droplets are nearly spherical. However, a feature of Leal's analysis was that the first, 0(Pe), flow-induced modifications of the local temperature field near a drop produced no contribution to the bulk conductivity, which only finally exhibited a flow-induced contribution at  $0(Pe^{3/2})$ . Since this result would appear to be due to the symmetry induced in the temperature field by the spherical shape of the drop, it is clearly possible that the dependence of the effective conductivity on Peclet number could be altered fundamentally when the shape is allowed to deviate from spherical; specifically, we may ask whether even very small deformations of shape might not cause the first order, 0(Pe), modifications of the local temperature field to make contributions to the bulk conductivity of the same or larger magnitude than the  $0(Pe^{3/2})$  contributions which were previously found by Leal.

In the present communication, we consider the case of small deformations of shape in the two limiting cases of dominant interfacial tension forces ( $\epsilon \sim a\gamma\mu/\sigma \ll 1$ ), and dominant internal (drop) viscosity effects ( $\epsilon \sim 1/\lambda \ll 1$ ). Here,  $\epsilon$  is the deformation parameter,  $\mu$  the viscosity of the suspending fluid,  $\lambda$  the ratio of internal to external viscosity, *a* the undeformed drop radius, and  $\sigma$  the interfacial tension. The case when interfacial tension and internal viscosity effects are the same order of magnitude is more difficult and will not be considered.

# The particle shape, local velocity field and temperature distribution

For small deformations of shape, the surface of a drop is most conveniently represented in the form originally suggested by Taylor (1932),

$$r_s = 1 + \epsilon f(\theta, \phi) + 0(\epsilon^2)$$
<sup>[1]</sup>

in which  $(r, \theta, \phi)$  is a spherical coordinate system with its origin at the geometric center of the drop. The radial coordinate has been nondimensionalized with respect to the undeformed drop radius, *a*. The precise nature of the deformation parameter  $\epsilon$ , which is assumed to be small, and the shape function  $f(\theta, \phi)$  depends on the physical limit which is considered. For the case of dominant interfacial tension forces, Taylor has shown

$$\epsilon = \frac{a\gamma\mu}{\sigma} \left( \frac{19\lambda + 16}{16\lambda + 16} \right), \quad f(\theta, \phi) = \sin^2 \theta \sin 2\phi.$$
 [2]

On the other hand, for dominant internal viscosity the corresponding results are (Taylor 1932)

$$\epsilon = \frac{5}{4\lambda}, \quad f(\theta, \phi) = \sin^2 \theta \cos 2\phi.$$
 [3]

In either case the creeping flow velocity fields outside and inside the drop may be represented by asymptotic expansions of the form

$$\mathbf{u} = \mathbf{u}^0 + \epsilon \, \mathbf{u}^1 + \mathbf{0}(\epsilon^2), \quad \bar{\mathbf{u}} = \bar{\mathbf{u}}^0 + \epsilon \, \bar{\mathbf{u}}^1 + \mathbf{0}(\epsilon^2), \tag{4}$$

in which all velocities are nondimensionalized with respect to  $\gamma a$ . The functions  $\mathbf{u}^{\circ}$  and  $\mathbf{\bar{u}}^{\circ}$  for a spherical drop, as well as the  $0(\epsilon)$  contributions in the dominant interfacial tension limit were obtained by Taylor (1932). The  $0(\epsilon)$  velocity fields in the dominant internal viscosity limit can be easily obtained using spherical harmonics of order 2 and 4 in the general solution of Lamb; for details see McMillen (1975).

In order to evaluate the effective conductivity, the local temperature fields must be obtained both inside and outside a drop. The governing equations, with temperatures nondimensionalized with respect to  $\bar{\alpha}a$ , are

$$\nabla^2 \bar{T} = Pe_2(\bar{\mathbf{u}} \cdot \nabla \bar{T}) \quad \text{(inside)},$$
[5]

$$\nabla^2 T = Pe_1(\mathbf{u} \cdot \nabla T) \quad \text{(outside)}, \tag{6}$$

where 
$$Pe_1 \equiv \frac{a^2 \gamma \rho C_{P_1}}{k_1}, \quad Pe_2 \equiv \frac{a^2 \gamma \rho C_{P_2}}{k_2}.$$

The subscripts 1 and 2 refer to the suspending fluid and the fluid in the drop respectively. Following the earlier analysis of Leal (1973), we shall assume that both  $Pe_1$  and  $Pe_2$  are small. Thus, the temperature distributions are calculated as perturbation expansions for the double limit  $\epsilon \ll 1$  and  $Pe_1$ ,  $Pe_2 \ll 1$ . The expansion in  $\epsilon$  is regular. However, at each order in  $\epsilon$ , the expansion in Pe is singular and most conveniently obtained by the method of matched asymptotic expansions, which was also employed by Leal (1973).

In the inner region, which includes the drop, the temperature distribution may be expressed in the form

$$T = f_0^{0}(Pe_1)T_0^{0} + f_1^{0}(Pe_1)T_1^{0} + f_2^{0}(Pe_1)T_2^{0} + \cdots$$
  
+  $\epsilon [f_0^{-1}(Pe_1)T_0^{-1} + f_1^{-1}(Pe_1)T_1^{-1} + f_2^{-1}(Pe_1)T_2^{-1} + \cdots] + 0(\epsilon^2),$  [7a]

$$\bar{T} = f_0^{\ 0}(Pe_2)\bar{T}_0^{\ 0} + f_1^{\ 0}(Pe_2)\bar{T}_1^{\ 0} + f_2^{\ 0}(Pe_2)\bar{T}_2^{\ 0} + \cdots$$
$$+ \epsilon [f_0^{\ 1}(Pe_2)\bar{T}_0^{\ 1} + f_1^{\ 1}(Pe_2)\bar{T}_1^{\ 1} + f_2^{\ 1}(Pe_2)\bar{T}_2^{\ 1} + \cdots] + 0(\epsilon^2),$$
[7b]

where the gauge functions  $f_i^j(Pe_1)$ , which must be found as part of the solution, satisfy the usual

relationship

$$\lim_{Pe_1\to 0}\frac{f_{n+1}^m}{f_n^m}\to 0.$$

The basic governing equations in this region, [5] and [6], are solved subject to the conditions of continuity of temperature and heat flux at the drop surface,

$$\bar{T}|_{r=r_s} = T|_{r=r_s},\tag{8}$$

$$k_1(\mathbf{n}\cdot\nabla\overline{T})|_{r=r_s} = k_2(\mathbf{n}\cdot\nabla T)|_{r=r_s},$$
[9]

plus boundedness at r = 0, and matching for large r with the solution in the outer region.

In the region far from the body, [6] must be rescaled in a manner consistent with the fact that the conduction and convection terms are of equal magnitude at large distances, even in the limit as  $Pe_1 \rightarrow 0$ . This requires the radial variable r to be rescaled according to  $\rho = rPe_1^{1/2}$ . For convenience,  $T^*$  is used to denote the temperature in this region. The exact form of the governing equation depends on the physical deformation limit which is considered. For the case of deformation dominated by interfacial tension forces

$$\nabla_{\rho}^{2}T^{*} - \hat{y}\frac{\partial T^{*}}{\partial \hat{x}} = Pe_{1}^{3/2}\left\{\frac{C}{4\rho^{2}}\sin^{2}\theta\sin2\phi + \epsilon\left[\frac{3A^{2}}{2\rho^{2}}\sin^{2}\theta\cos2\phi + \frac{5A_{-3}}{14\rho^{2}}(2-3\sin^{2}\theta)\right]\right\}\frac{\partial T^{*}}{\partial\rho} + 0(Pe_{1}^{5/2}),$$
[10]

where  $(\hat{x}, \hat{y}) = [Pe_1^{1/2}x, Pe_1^{1/2}y], \nabla_{\rho}^2$  represents the usual Laplacian operator with  $\rho$  replacing *r*, and *C*,  $A_{-3}^2$  and  $A_{-3}$  are coefficients given by Chaffey, Brenner & Mason (1965) as:

$$A_{-3}^2 = \frac{19\lambda + 16}{15\lambda + 15}, \quad A_{-3} = \frac{25\lambda^2 + 41\lambda + 4}{25(\lambda + 1)^2}, \quad C = -\frac{(2+5\lambda)}{\lambda + 1}$$

For the case of deformation dominated by internal viscous forces, we obtain

$$\nabla_{\rho}^{2}T^{*} - \hat{y}\frac{\partial T^{*}}{\partial \hat{x}} = Pe_{1}^{3/2}\left\{\frac{C}{4\rho^{2}}\sin^{2}\theta\,\sin 2\phi\,\frac{\partial T^{*}}{\partial\rho}\right\} + 0(Pe_{1}^{5/2}).$$
[11]

In this outer region, an expansion for  $T^*$  similar to [7] is used, and the resulting equations at each order solved subject to the condition

$$T^* \rightarrow \frac{\alpha \rho \sin \theta \sin \phi}{P e_1^{1/2}} \text{ as } \rho \rightarrow \infty,$$
 [12]

along with the matching condition

$$\lim_{\rho \to 0} T^*(\rho, \theta, \phi) \Leftrightarrow \lim_{r \to \infty} T(r, \theta, \phi) \text{ as } Pe_1 \to 0.$$
 [13]

At first order in  $\epsilon$ ,  $T_i^0$ ,  $\bar{T}_i^0$ , and  $T_i^{*0}$  are simply the temperature fields for a perfectly spherical drop which were evaluated previously by Leal (1973) up to  $T_2^0$ ,  $\bar{T}_2^0$ , and  $T_2^{*0}$ . However, the two solutions  $T_1^0$  and  $\bar{T}_1^0$  were slightly in error in the earlier work, and have been corrected in the present study. For the sake of brevity, we omit details of the solutions from the present paper and refer the interested reader to McMillen (1975). It need only be noted here that the first deformation-induced flow contributions to the effective conductivity arise from the temperature correction of order  $\epsilon Pe_1$  in the case of deformation dominated by interfacial tension forces. For the case of deformation dominated by internal viscosity, however, the order  $\epsilon Pe_1$  term produces *no* contribution to the effective (bulk) conductivity, and terms through  $0(\epsilon Pe^{3/2})$  are required in the inner expansions for the temperature.

We now turn to our main objective, namely the calculation of the first deformation-induced flow contributions to the effective conductivity of a dilute suspension of slightly deformed drops.

## Calculation of effective conductivity from microscale fields

A general expression for the effective conductivity of a dilute suspension of identical particles was obtained by Leal (1973) for heat transfer across a linear bulk shear flow. For the case of slightly deformed particles this expression is

$$k_{\rm eff} = k_1 - Q_i'/\alpha, \qquad [14]$$

where in terms of nondimensional quantities

$$Q'_{i} = \frac{3(k_{1}-k_{2})\Phi}{4\pi} \int_{0}^{2\pi} \int_{0}^{\pi} (n_{y}T)_{r-r_{s}} r_{s}^{2} \sin \theta \partial \theta \partial \phi + \frac{3k_{1}}{4\pi} P e_{1} \Phi \int_{0}^{2\pi} \int_{0}^{\pi} \int_{r_{s}}^{\infty} u'_{y}T' r^{2} \sin \theta \partial r \partial \theta \partial \phi$$
$$+ \frac{3k_{2}}{4\pi} P e_{2} \Phi \int_{0}^{2\pi} \int_{0}^{\pi} \int_{0}^{r_{s}} \bar{u}'_{y}\bar{T}' r^{2} \sin \theta \partial r \partial \theta \partial \phi.$$

Here,  $\Phi$  is the volume fraction of suspended particles,  $u'_y$  and T' are the disturbance velocity and temperature fields,  $u'_y = u_y$ ,  $T' = T - \alpha y$ , and  $n_y$  is the j component of the unit outer normal to the particle surface. In the present work, the asymptotic expressions for T,  $\overline{T}$ ,  $T^*$ , u,  $\overline{u}$ , and  $n_y$  in terms of  $\epsilon$  and *Pe* are used to evaluate this expression for the effective conductivity. For the case of surface tension controlling deformation, we obtain

$$\frac{k_{\text{eff}}}{k_1} = k_{\text{eff}}^* = 1 + \Phi \left\{ \frac{3(m-1)}{m+2} + \left( \frac{1.176(m-1)^2}{(m+2)^2} + \frac{5\lambda+2}{\lambda+1} \left( 0.12 \left[ \frac{5\lambda+2}{\lambda+1} \right] - 0.028 \left[ \frac{m-1}{m+2} \right] \right) \right) P e_1^{3/2} + I(m,\lambda,\tau) \epsilon P e_1 + 0(\epsilon^2) + 0(P e_1^{-2}) + 0(\epsilon P e_1^{-3/2}) + \cdots \right\},$$
[15]

where *m* is the ratio of internal to external conductivities  $k_2/k_1$ , and  $\tau$  is the ratio of internal to external heat capacities  $Cp_2/Cp_1$ .  $I(m, \lambda, \tau)$  is a rather complex function of the three physical property ratios  $m, \lambda, \tau$  which is given explicitly in table 1.

For the case of internal viscous forces controlling deformation we find

$$k_{\text{eff}}^{*} = 1 + \Phi \left\{ \frac{3(m-1)}{m+2} + \left( 3.00 - 0.14 \frac{(m-1)}{m+2} + 1.76 \frac{(m-1)^{2}}{(m+2)^{2}} \right) P e_{1}^{3/2} - \left( 3.6 \frac{(m-1)^{2}}{(m+2)^{2}} \right) \epsilon + \left( 1.411 \frac{m(m-1)^{2}}{(m+2)^{3}} \left[ \left( \frac{\tau}{m} \right)^{3/2} - 1 \right] - 0.168 \frac{(m-1)^{2}}{(m+2)^{2}} \right] \epsilon P e_{1}^{3/2} + 0(\epsilon^{2}) + 0(P e_{1}^{2}) + 0(\epsilon P e_{1}^{2}) + \cdots \right\}.$$
[16]

In each case, the 0(1) and  $0(Pe_1^{3/2})$  terms arise from Leal's (1973) calculation for a spherical drop. The new terms arising due to drop deformation are of order  $\epsilon$  and  $\epsilon Pe_1^{3/2}$  for the case of deformation dominated by internal viscosity. Thus, in general, the deformation induced contributions in this limit represent small corrections, which are monopolized by the 0(1) and  $0(Pe_1^{3/2})$  terms of a perfect sphere. In contrast, however, the new term for the case of deformation dominated by interfacial tension is of order  $\epsilon Pe_1$ . In this case, the relative magnitudes of the flow-induced contribution for a

Table 1.  $\theta(\epsilon P_1)$  Contribution to the effective conductivity for deformation controlled by surface tension forces

$$\begin{split} I(m,\lambda,\tau) &= \frac{1}{(3m+4)(m+2)^3m(\lambda+1)^2} \times \{\tau\{[0.36m^5+0.48m^4-1.08m^3-0.72m^2+0.96m]\lambda^3 \\ &+ [-0.66m^5-1.023m^4+5.539m^3+10.956m^2+4.479m-8.773]\lambda^2 \\ &+ [-2.652m^5-6.561m^4+11.522m^3+27.381m^2-4.570m-20.002]\lambda \\ &+ [-1.604m^5+14.370m^4+50.410m^3+23.103m^2-61.640m-27.514]\} \\ &+ [2.273m^5-0.534m^4-41.673m^3+33.756m^2+23.754m+1.341]\lambda^2 \\ &+ [-1.110m^5-3.035m^4-37.122m^3+10.255m^2+20.050m+7.096]\lambda \\ &+ [-5.188m^5+20.813m^4+6.986m^3-48.044m^2-0.289m+2.940]\} \end{split}$$

perfect sphere which is  $0(Pe_1^{3/2})$ , and the deformation-induced flow contribution of  $0(\epsilon Pe_1)$  depend critically on  $\epsilon$  and  $Pe_1$ . The difference in the two small deformation limits is apparently a reflection of differences in the nature of the particle shape in the two cases. In the first case, the drop deforms with its axis of elongation along the x-axis and its axis of contraction along the y-direction. In the second case, the drop deforms along the principal axis of strain of the undisturbed shear flow, thus elongating along an axis 45° counterclockwise from the positive x-axis and contracting along the perpendicular axis which is 45° from the positive y-axis. Let us now examine the results, [15] and [16], in more detail.

For the case of deformation dominated by internal viscous effects, we have plotted in figure 1 the magnitude of the deformation-induced convective contribution to the effective conductivity as a function of the conductivity ratio m for several values of the heat capacity ratio  $\tau$ , i.e. the last term in [16]. Although the behavior of this term may seem to depend strongly on  $\tau$ , it is positive in all cases for m sufficiently small, negative when m is large, and zero at m = 1. In fact, for  $m \to \infty$ , the effective conductivity,  $k_{eff}^*$ , asymptotically approaches the value

$$k_{\text{eff}}^* \sim 1 + \Phi\{3 + 4.13Pe_1^{3/2} - 3.6\epsilon - 1.579\epsilon Pe_1^{3/2}\}, \quad (m \to \infty),$$
<sup>[17]</sup>

for arbitrary, fixed  $\tau$ . In this limit, both deformation-induced terms are small corrections to the dominant terms which correspond to a strictly spherical drop. It should be noted that the limit  $m \to \infty$ 

4.000



Figure 1. The  $0(\epsilon P e_1^{3/2})$  term in the effective conductivity when viscous forces control, as a function of the conductivity ratio,  $m = k_2/k_1$ , for several values of the heat capacity ratio,  $\tau = Cp_2/Cp_1$ . (a)  $\tau = 3$ , (b)  $\tau = 2$ ; (c)  $\tau = 1$ ; (d)  $\tau = 0.5$ ; (e)  $\tau = 0$ .

must be applied for small, fixed values of  $Pe_1$  in order that both  $Pe_1$  and  $Pe_2$  remain small as required by our basic solution for the temperature field. The limit  $m \rightarrow 0$  must be taken with  $Pe_2$  small and fixed for the same reason. In this limit

$$k_{\text{eff}}^* \sim 1 + \Phi \left\{ -\frac{3}{2} + 3.36 \frac{m^{3/2}}{\tau^{3/2}} P e_2^{3/2} - 0.9\epsilon + 0.176 m\epsilon P e_2^{3/2} \right\}, \quad (m \to 0).$$
 [18]

and both of the convective corrections are positive and vanishingly small, the deformation induced correction vanishing more slowly,  $0(m\epsilon Pe_2^{3/2})$ , than the convective correction for a spherical drop,  $0(m^{3/2}Pe_2^{3/2})$ , for  $\epsilon$  and  $Pe_2$  both small but of fixed value. Finally, when m = 1, the conductivities of the two fluids are equal and the particle contribution to the effective conductivity is produced entirely by the convective action of the fluid. For this case,

$$k_{\text{eff}}^* = 1 + \Phi\{3Pe_1^{3/2} + 0(\epsilon^2) + 0(Pe_1^2) + 0(\epsilon Pe_1^2) + \cdots\}, \quad (m = 1).$$
[19]

Thus not only do the pure conduction contributions to  $k_{eff}^*$  vanish, as expected, when the thermal conductivities are equal, but, surprisingly, the  $0(\epsilon P e_1^{3/2})$  deformation-induced convective term also vanishes. These results, [17]–[19], would seem to imply that the deformation contributions in the dominant internal viscosity case are always small when compared to those for a perfectly spherical



Figure 2. The  $0(\epsilon Pe_1)$  term,  $I(m, \lambda, \tau)$ , in the effective conductivity when interfacial tension forces control, as a function of the conductivity ratio,  $m = k_2/k_1$ , and the viscosity ratio,  $\lambda = \mu_2/\mu_1$ , for several values of the heat capacity ratio,  $\tau = Cp_2/Cp_1$ . Detail a:  $\tau = 0.5$ . Detail b:  $\tau = 1$ . Detail c:  $\tau = 2$ .

drop, except for  $m \rightarrow 0$  when both are very small. When the internal and external conductivities are of similar magnitude, the deformation-induced flow contribution actually vanishes altogether.

For the case of deformation controlled by surface tension forces, the dominant deformation-induced correction,  $0(\epsilon Pe_1)$ , can be either positive or negative depending on the values of the three ratios,  $m, \lambda$  and  $\tau$ . The general characteristics of the function  $I(m, \lambda, \tau)$  are demonstrated graphically in figure 2 where  $I(m, \lambda, \tau)$  is plotted as a function of m and  $\lambda$  for several values of  $\tau$  in the range  $\lambda \leq 0(1)$  which is required in order that [2] and [4] remain valid. The three limiting cases m = 1,  $m \to \infty$ , and  $m \to 0$  are of special interest, and can again be evaluated analytically. When m = 1,

$$k_{\text{eff}}^* = 1 + \Phi \left\{ 0.12 \frac{(5\lambda + 2)^2}{(\lambda + 1)^2} P e_1^{3/2} + [(0.055\lambda^2 + 0.027\lambda - 0.015)\tau + (0.099\lambda^2 - 0.020\lambda - 0.120)] \frac{\epsilon P e_1}{(\lambda + 1)^2} \right\}.$$
[20]

In the limit as  $m \rightarrow \infty$ , the effective conductivity has the asymptotic form

$$k_{\text{eff}}^{*} = 1 + \Phi \left\{ 3 + \left[ 1.176 + \frac{5\lambda + 2}{\lambda + 1} \left( 0.12 \left( \frac{5\lambda + 2}{\lambda + 1} \right) - 0.028 \right) \right] P e_{1}^{3/2} + \epsilon P e_{1} \frac{1}{(\lambda + 1)^{2}} \right. \\ \left. \left. \left\{ (0.12\lambda^{3} - 0.22\lambda^{2} - 0.884\lambda - 0.534)\tau + 2.273\lambda^{2} - 1.110\lambda - 5.188 + \cdots \right\}, \quad m \to \infty.$$

The main point of interest with regard to both [20] and [21] is the fact that the flow-induced corrections  $0(Pe_1^{3/2})$  and  $0(\epsilon Pe_1)$  are both nonzero for  $\lambda < 0(1)$ . Hence, as in the general case, the relative importance of the deformation-induced term and the term for an exact sphere depends on the magnitudes of  $\epsilon$  and  $Pe_1$ . Finally, we turn to the limiting case  $m \to 0$ . It is evident from figure 2 that  $I(m, \lambda, \tau)$  is very strongly negative in this limit for  $0.5 < \tau < 2$ . Indeed, for  $m \to 0$ ,  $I(m, \lambda, \tau) \sim f(\lambda, \tau)/m$ . Provided care is taken to hold  $Pe_2$  constant (and small), however, the effective conductivity can be seen to have the asymptotic form

$$k_{\text{eff}}^{*} = 1 + \Phi \left\{ -\frac{3}{2} + \left[ (-8.773\lambda^{2} - 20.002\lambda - 27.514) + \frac{1}{\tau} (1.341\lambda^{2} + 7.096\lambda + 2.940) \right] \frac{\epsilon P e_{2}}{32(\lambda + 1)^{2}} + \cdots \right\} \quad (m \to 0).$$
[22]

It is particularly noteworthy that in this limit the deformation-induced flow contribution to  $k_{eff}^*$  completely dominates the largest flow contribution which occurs for a spherical drop. Thus, as we suggested in the introduction, the presence of even a small degree of shape deformation can cause a fundamental change in the nature of the dominant flow contribution to the effective thermal conductivity. Although the corrections in the present analysis are small in any case due to the assumptions of small  $\Phi$ ,  $Pe_1$ ,  $Pe_2$  and  $\epsilon$ , it would appear that care must be taken in attempting to correlate experimental data for any suspension in which the particles are not exactly spherical with theoretical results for spherical particles.

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**Résumé**—On calcule la conductibilité thermique effective d'une suspension diluée de gouttelettes légèrement déformées, dans le cas limite où elles ont un faible nombre de Péclet, en prenant, pour le milieu, un effort de cisaillement non perturbé,  $u = \gamma y$ , et un gradient de température constant  $T = \alpha y$ . On considère deux cas distincts de petites déformations; déformation dominée par les forces de tension interfaciale, et déformation dominée par les forces de tension interfaciale, et déformation peut provoquer un changement fondamental dans la forme de la contribution dominante, induite par l'écoulement, à la conductibilité effective.

Auszug-Die effektive Waermeleitfaehigkeit einer verduennten Suspension von schwach verformten Tropfen wurde berechnet, an der Grenzekleiner Teilchen-Péclet-Zahlen, fuer ungestoerte Volumgleitung  $u = \gamma y$  und linearen Volumtemperaturgradienten  $T = \alpha y$ . Zwei unterschiedliche Faelle kleiner Verformung werden betrachtet, naemlich durch Zugkraefte in der Grenzflaeche, und durch Viskositaetskraefte im Tropfen bestimmte Verformungen. Die Ergebnisse zeigen, dass das Auftreten einer Deformation eine grundsaetzliche Aenderung in Bezug auf den vorherrschenden stroemungsinduzierten Beitrag zur effektiven Leitfaehigkeit hervorrufen kann.

Резюме—Эффективная теплопроводность разведенных суспензий незначительно деформированных капелек подсчитана при ограничениим небольшими значениями коитерия Пекле ненарушенного профиля основной массы  $\mu = \gamma$ . У и линейного градиента температур в ней  $T = \alpha$ . У. Рассмотрены два различных случая малых деформаций: дефармация, определяемая силами поверхностного натяжения между фазами, и деформация, определяемая силами вязкости в капле. Результаты показывают, что наличие деформации может привести к существенным изменениям в форме ведущего вклада, вызываемого течением, в эффективную теплопровдность.